The Role of Visualization in Mathematics Education:

Can visualization Promote the Causal Thinking?

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I would rather discover one cause than gain the kingdom of Persia.
Democritus (ca. 460 B.C.--370 B.C.)

1. Concepts' and relations' visualisation in today's use

In classroom activities, as in textbooks, we do use visualisation to visualise mathematical concepts and relations. There are quite common problems in this use. In some cases visualisation do not allow children to make generalisation of relationships. In other cases, it could present a relation, but it does not reveal the reason of its existence. In other words it misleads children and makes them jump to conclusions, which have not been justified. The most common problem is that visualisation use offers in a static way ready model and leave a little to students for searching for the cause.

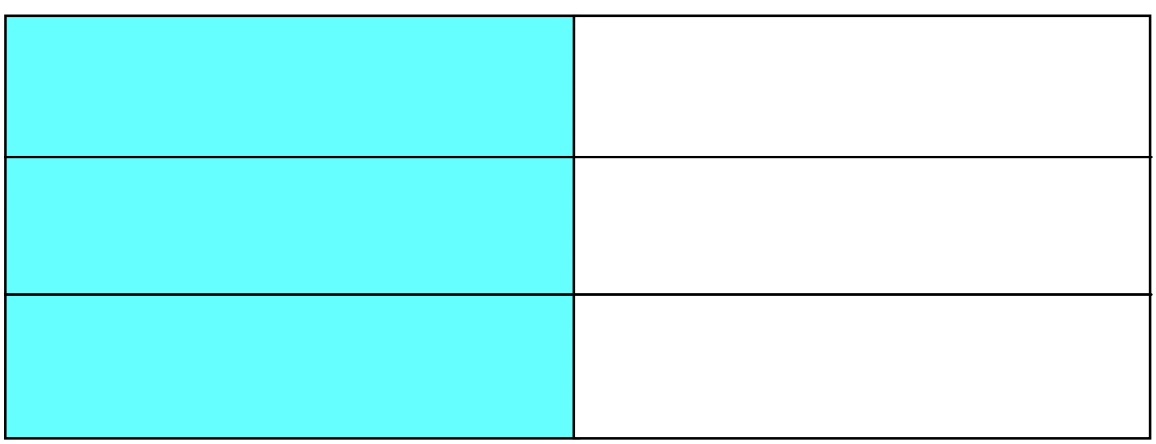
2. An example of the problems

Eqality of fractions is an essential part of primary school mathematics. In Finland, as in some other countries, multiplying a fraction terms by a positive integer to get an equal fraction is an operation and has a special term 'expansion'. We do even use special notation in fulfilling this operation.

For instance, when we expand the fraction $\frac{1}{2}$ by 3 to get the equal fraction $\frac{3}{6}$ we write this performance in the next form:

$$^{3)}\frac{1}{2} = \frac{3}{6}$$

The common type of visualisation to justify the truth of this proportion is the next:



(Figure 1)

Indeed this figure shows that $\frac{1}{2} = \frac{3}{6}$, but it has two pedagogical problems. The first is that this figure by itself does not offer a reasonable cause of multiplying the terms of the fraction $\frac{1}{2}$ by 3. The other problem is that we can not rely on such figure, of a case, to make a generalisation, and saying that multiplying the terms of a fraction by a positive integer gives an equal fraction. In the next chapter we shall discuss in more details the first problem.

3. The problem of causality

To explain why figure 1 does not justify by itself the multiplication of fraction $\frac{1}{2}$ terms by three to get the fraction $\frac{3}{6}$, let us discuss the next situation.

Assume that we use figure 1 to come to conclusion that $\frac{1}{2}=\frac{3}{6}$ and explaining how we can proceed from $\frac{1}{2}$ to get $\frac{3}{6}$ as follows.

$$^{3)}\frac{1}{2} = \frac{3^{1}}{3^{2}-3} = \frac{3}{9-3} = \frac{3}{6}$$

In other words we can use number 3 to obtain the fraction $\frac{3}{6}$ from the fraction $\frac{1}{2}$ by different means than multiplication both terms of fraction $\frac{1}{2}$ terms by 3.

4. Towards better use of visualisation

In Finland, as in many other countries, we are making different attempt to develop the using of visualisation. We do develop our type of visualisation and test it by different means, among others teaching in our university mathematical clubs. One of other means to be mentioned is the Pre-service and in-service education of teachers, among them textbooks' authors.

The main characters of our type of visualisation are the next:

1) The figure used for visualisation a mathematical entity has to be as simple as possible.
2) Figure must be isomorphic to the visualised mathematical entity.

Having this two characters means that visualisation can allow to the next two improvements of using visualisation:

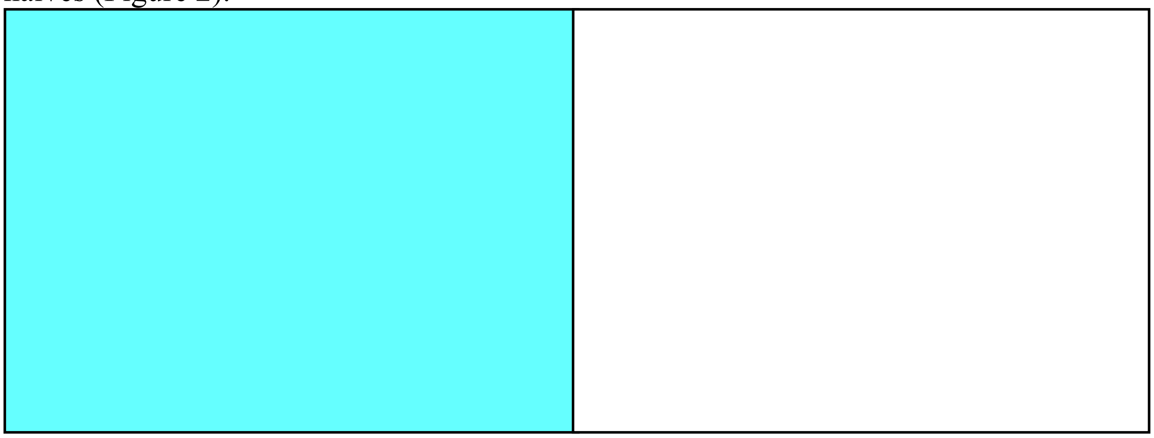
1) Visualisation is a way to understand mathematical principles in a more general form.
2) Visualisation is a way to motivate children to search for the cause of the truth of a principle.

These last two improvements mean that visualisation is not for demonstrating justification of a principle, but for motivating to search for this justification.

5. An improvements' example

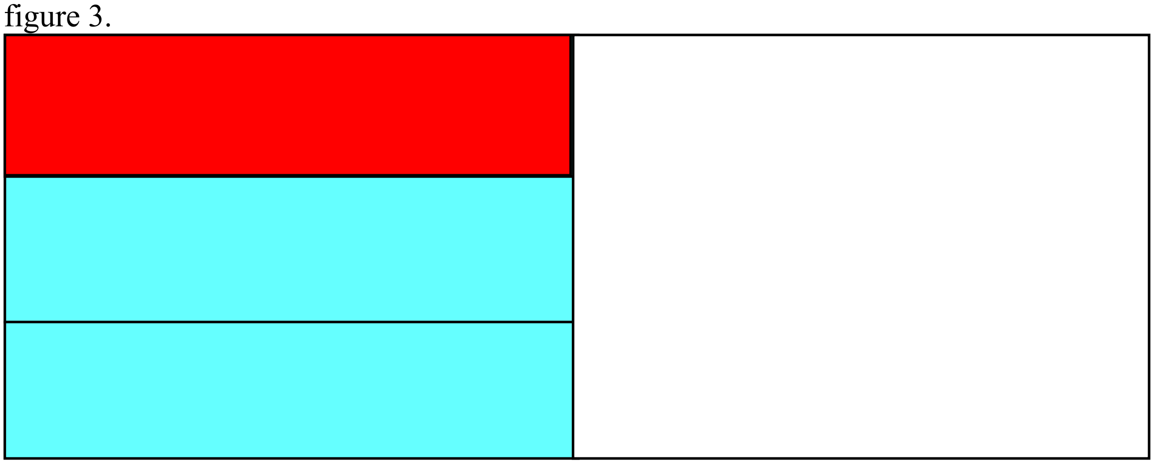
Let us take as an example the improvements we have made in visualisation to teach fraction 'expansion' principle.

We first draw a rectangle to represent a whole. After that we ask children to use a segment to divide the rectangle into two equivalent rectangles and colour one of the two halves (Figure 2).



(Figure 2)

Now we ask children to determine what is the unit fraction we deal with, and how many of them we have in Figure 2. We ask them to take attention to the blue rectangle as a representative of our unit fraction $\frac{1}{2}$. After that we divide the blue rectangle into three equivalent rectangles and after while colouring one of these three rectangles red as in figure 3.

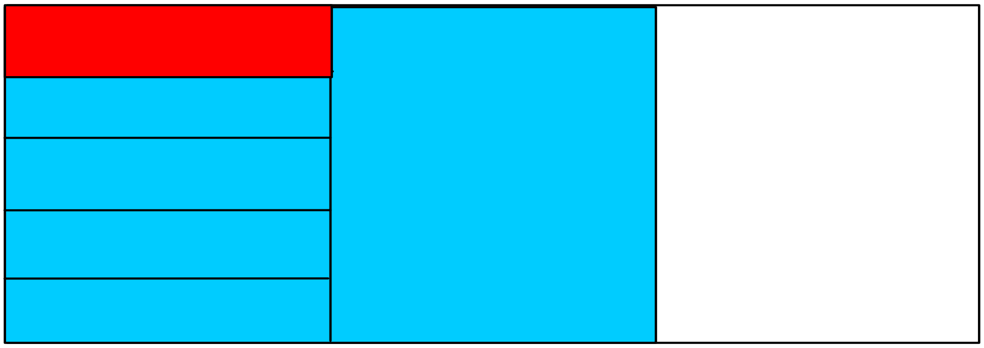


(Figure 3)

Now we ask children to take the red triangle as our new unit fraction and ask them to tell us how many of such new unit fractions we have. Then we came to the **decisive question:** How many times of such three units we should have in the whole rectangle? This is a time to discuss the open sentence $\frac{1}{2} = \frac{? \times 3}{? \times 3}$, in which number three we coloure in red.

We do ask the children to tell us what is the new unit fraction we have, how many of them we have in a half, and how many of them to have in a whole.

To demonstrate the positive sides of our way of visualisation let us discuss in brief the next figure:



(Figure 4)

We can use this figure to find out the numbers needed to replace the question marks in the next open sentence: ${}^{5)}\frac{2}{3} = \frac{? \times 5}{? \times ?}$. This visualisation permits children to understand that we are changing the unit fraction from $\frac{1}{3}$ to $\frac{1}{15}$. Figure 4 visualise clearly the multiplication factors: the ***multiplicand*** and ***multiplier***. On contrast, dividing the whole rectangle in figure 4 into 15 equivalent rectangles, in a similar way to the case of figure 1, does not help in seeing the idea behind multiplying the terms of $\frac{2}{3}$ by 5.

6. Conclusion

The example discussed in this short paper allows us to show how visualisation could be a tool to motivate children to search for the cause of principles and be able to make generalisation. It is not a static demonstration of facts, but a tool to develop the causal thinking or say the habit for searching for the cause. It is a type of dynamic interactive visualisation, which among others allow developing visual imagery.

Literature (English published related)

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